

Statistics

Lecture 22



Feb 19-8:47 AM

I surveyed 175 students and 24% of them were in favor of online classes.

1) How many of them were in favor of online classes? $n=175$, $\hat{P}=.24$

$$x = n\hat{p} = 175(.24) = 42$$

if decimal, Round-up

2) Find Conf. interval for the proportion of all students that are in favor of online classes

\rightarrow **No C-level $\rightarrow .95$**

1 - Prop \neq Int

$$.18 < P < .30$$

3) Find the margin of error.

$$E = \frac{.30 - .18}{2} = .06$$

Nov 18-12:14 PM

A sample of 28 students had a mean test score of 76.5. $n=28, \bar{x}=76.5$

It is known that standard deviation of all test scores is 9.5. $\sigma=9.5$

1) Find 96% Conf. interval for the mean score of all exams.

σ known $\Rightarrow Z$ Interval

Since \bar{x} is 1-decimal $\rightarrow 72.8 < \mu < 80.2$

2) Find margin of error $E = \frac{80.2 - 72.8}{2} = 3.7$

3) Find $Z_{\alpha/2}$ or $t_{\alpha/2}$ for this problem. $\boxed{3.7}$

σ known $\Rightarrow Z_{\alpha/2}$

$Z_{\alpha/2} = \text{invNorm}(.98, 0, 1) = 2.054$

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I randomly selected 12 cars on a certain FWY. Here are their speed.

75	82	68	80
70	58	60	78
74	85	55	70

Store in L1
Use 1-Var stats

1) $\bar{x} \approx 71$
2) $s \approx 10$ } Round to whole #

3) Find Conf. interval for the mean speed of all cars on that FWY.

σ unknown $\Rightarrow T$ Interval

NO C-level $\Rightarrow .95$

Since \bar{x} is a whole $\rightarrow 65 < \mu < 77$

4) Find margin of error $E = \frac{77 - 65}{2} = 6$

5) Find $Z_{\alpha/2}$ or $t_{\alpha/2}$ for this example.

σ unknown $\Rightarrow t_{\alpha/2}$

$t_{\alpha/2} = \text{invT}(.975, 11) = 2.201$

σ unknown
 $df = n - 1 = 11$

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How to determine minimum Sample Size n :

Population Proportion

$$E = Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}} \quad \text{with some algebra work}$$

$$n = \hat{p}\hat{q} \left(\frac{Z_{\alpha/2}}{E} \right)^2$$

If decimal,
Round-up

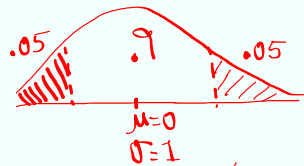
If \hat{p} & \hat{q} are both unknown, use .5 for each

$$n = .25 \left(\frac{Z_{\alpha/2}}{E} \right)^2$$

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Find minimum sample size needed to
Construct 90% Conf. interval for Pop.
Proportion and error not to exceed 8%.

1) Assume $\hat{p} = .3$.



$$Z_{\alpha/2} = \text{invNorm}(.95, 0, 1) = 1.645$$

$$n = \hat{p}\hat{q} \left(\frac{Z_{\alpha/2}}{E} \right)^2$$

$$= (.3)(.7) \left(\frac{1.645}{.08} \right)^2$$

$$= 88.791 \dots$$

$$n = 89$$

2) \hat{p} & \hat{q} are both unknown.

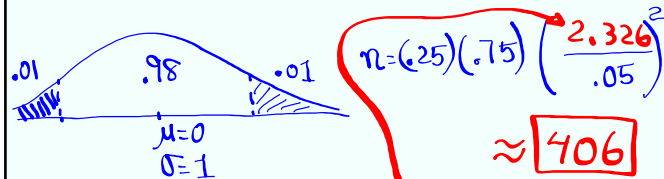
$$n = .25 \left(\frac{Z_{\alpha/2}}{E} \right)^2 = .25 \left(\frac{1.645}{.08} \right)^2 = 105.704$$

$$n \approx 106$$

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Find min. Sample Size needed to Construct
98% Conf. interval for pop. prop. and
 error not exceed 5%.

1) Assume $\hat{p} = .25$ $n = \hat{p}\hat{q} \left(\frac{Z_{\alpha/2}}{E} \right)^2$



$Z_{\alpha/2} = \text{inv Norm}(.99, 0, 1) = 2.326$

2) \hat{p} & \hat{q} are unknown. $n = .25 \left(\frac{Z_{\alpha/2}}{E} \right)^2$
 $= .25 \left(\frac{2.326}{.05} \right)^2$
 ≈ 406

Nov 18-12:56 PM

How to determine minimum Sample Size n :

Population Mean

$E = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$ with some algebra work

$$n = \left(\frac{Z_{\alpha/2} \cdot \sigma}{E} \right)^2$$

If decimal,
Round-up

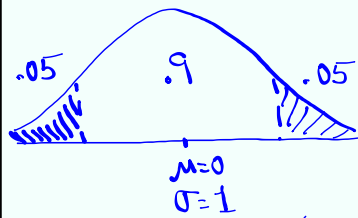
If σ is unknown \rightarrow use S instead

$$n = \left(\frac{Z_{\alpha/2} \cdot S}{E} \right)^2$$

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Given C-level: .9, $\sigma = 12.5$, $E = 5$

Find minimum Sample Size n
for Constructing C.I. for Pop. Mean.



$$Z_{\alpha/2} = \text{invNorm}(.95, 0, 1) = 1.645$$

$$n = \left(\frac{Z_{\alpha/2} \cdot \sigma}{E} \right)^2$$

$$= \left(\frac{1.645 \cdot 12.5}{5} \right)^2$$

$$n \approx 17$$

Redo with $E = 2.5$

$$n = \left(\frac{1.645 \cdot 12.5}{2.5} \right)^2 \approx 68$$

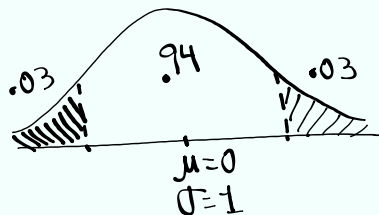
Nov 18-1:07 PM

Given: C-level: .94 $S = 14$

$$E = 6$$

Find min. Sample Size n to Construct
C.I. for Pop. Mean.

σ Unknown \rightarrow use S in place of σ .



$$Z_{\alpha/2} = \text{invNorm}(.97, 0, 1) = 1.881$$

$$n = \left(\frac{Z_{\alpha/2} \cdot S}{E} \right)^2$$

$$= \left(\frac{1.881 \cdot 14}{6} \right)^2 \approx 20$$

Nov 18-1:13 PM

In a sample of 285 people in LA,
73% of them were fan of Dodgers.

1) Find Conf. interval for the prop. of all
people in LA that are fan of Dodgers.

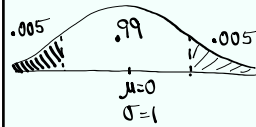
$$n = 285 \Rightarrow x = n\hat{p} = 285(.73) = 208.05 \quad [x = 209]$$

No C-level $\Rightarrow .95$ 1-Prop Z Int

$$.68 < P < .78$$

$$E = .05$$

2) Find min. Sample Size needed to Construct
99% Conf. interval for prop. of all with
error not to exceed 4%.



$$n = \hat{p}\hat{q} \left(\frac{Z_{\alpha/2}}{E} \right)^2$$

$$= (.73)(.27) \left(\frac{2.576}{.04} \right)^2$$

$$Z_{\alpha/2} = \text{invNorm}(.995, 0, 1) = 2.576 \quad [n = 818]$$

Assume \hat{p} & \hat{q} are both unknown

$$n = .25 \left(\frac{2.576}{.04} \right)^2 \approx 1037$$

Nov 18-1:40 PM

I randomly selected 10 Cars on FWY 60.
Here are their speed

65	80	75	78
68	82	72	62
58	79		

Store in L1

Use 1-Var Stats

Find

$$\bar{x} \approx 72$$

Round
to
whole

$$s \approx 8$$

Find 88% Conf. interval
for mean speed of all Cars on FWY 60.

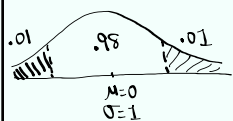
σ unknown \Rightarrow T Interval

$$68 < \mu < 76$$

$$E = \frac{76 - 68}{2} = 4$$

Find min. Sample Size needed

to Construct 95% Conf. interval for mean
speed of all Cars on FWY 60 and margin
of error not to exceed 5 mph.



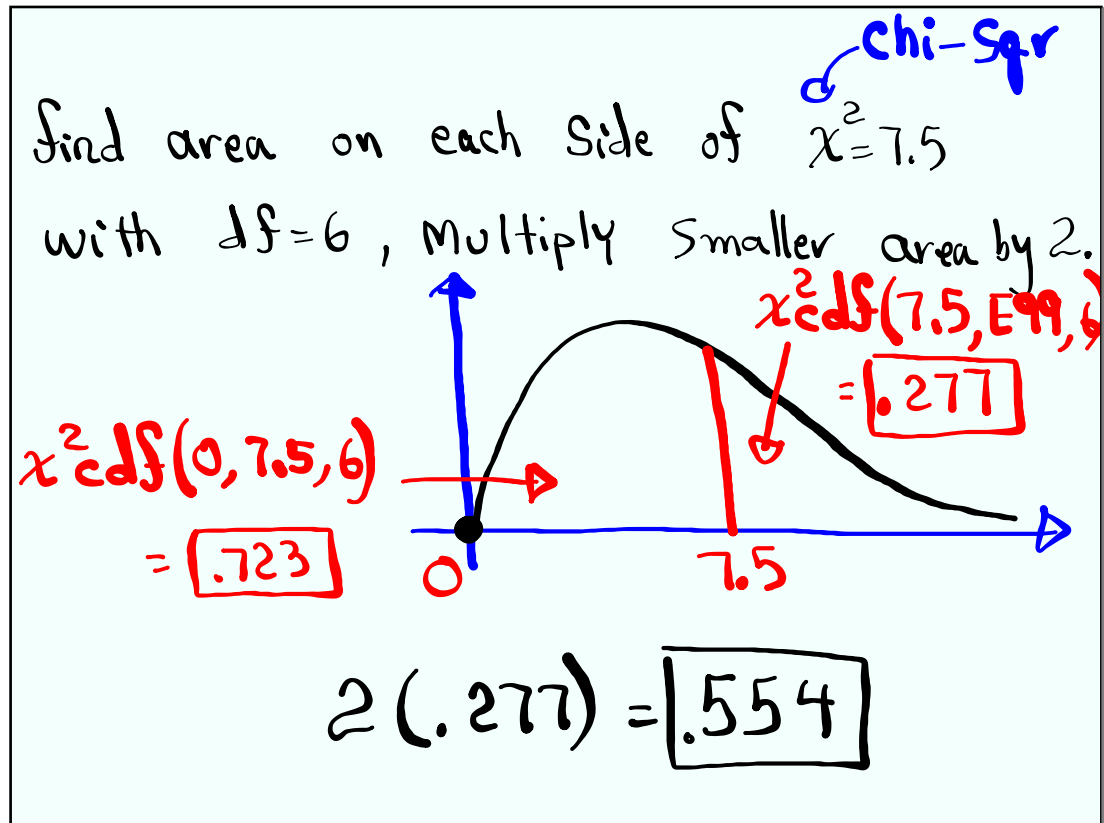
$$n = \left(\frac{Z_{\alpha/2} \cdot \sigma}{E} \right)^2$$

$$Z_{\alpha/2} = \text{invNorm}(.975, 0, 1) = 2.326$$

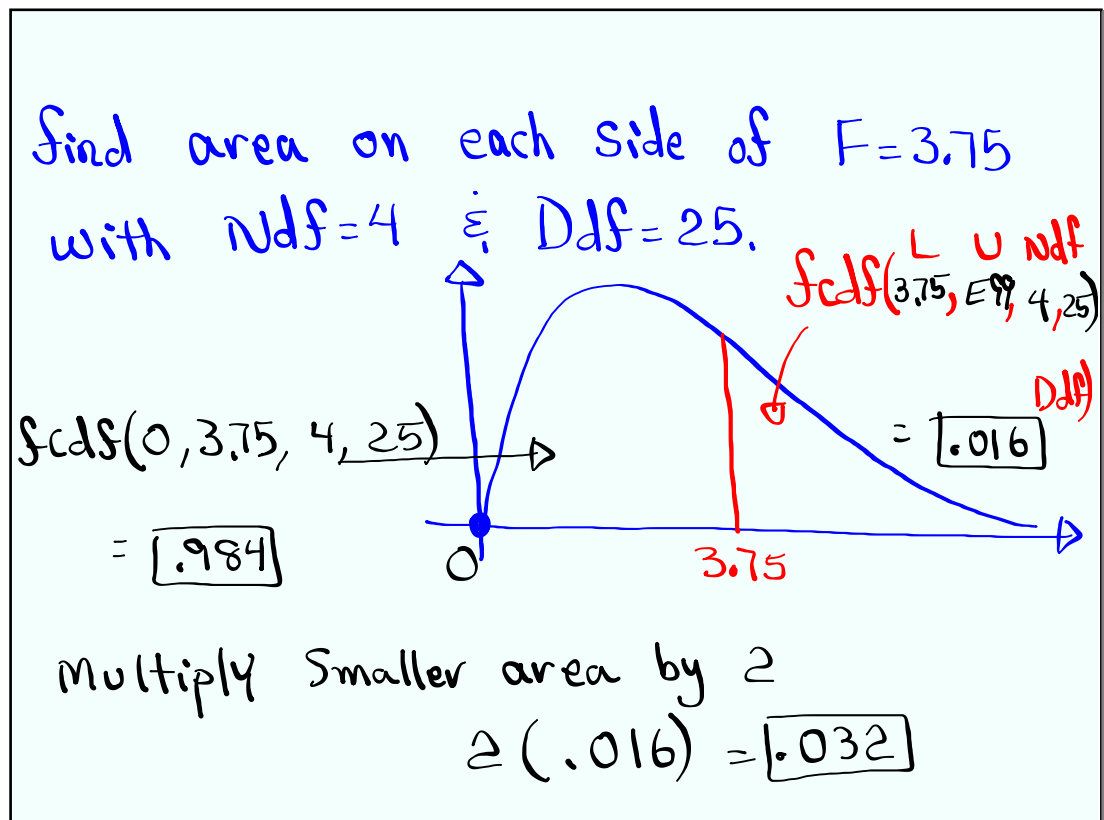
$$= \left(\frac{2.326 \cdot 8}{5} \right)^2$$

$$\approx 14 \text{ Cars}$$

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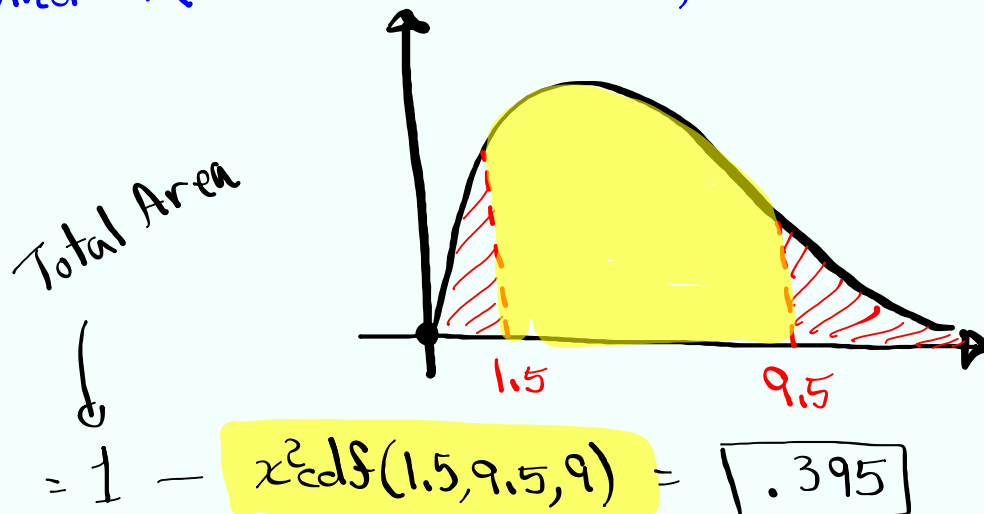


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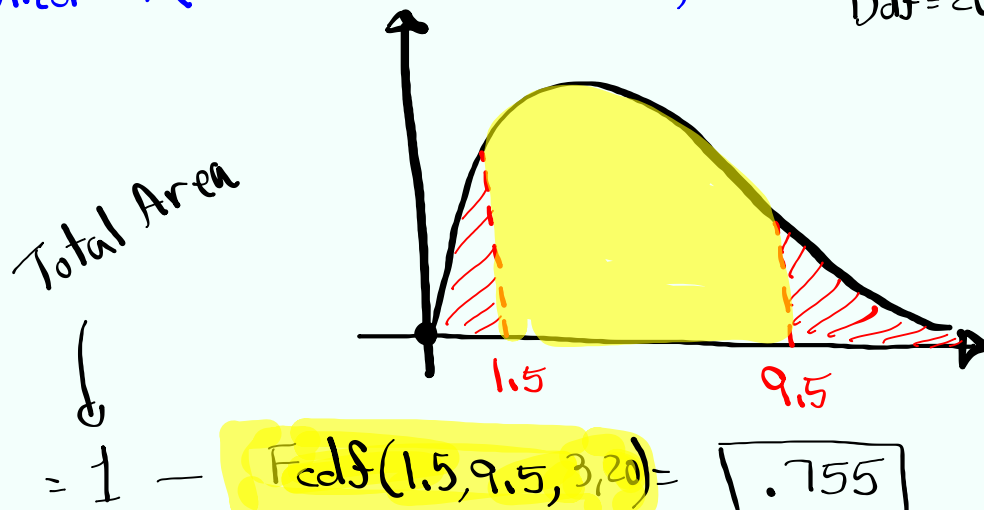
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find $P(\chi^2 < 1.5 \text{ OR } \chi^2 > 9.5)$ with $df=9$.



Nov 18-2:19 PM

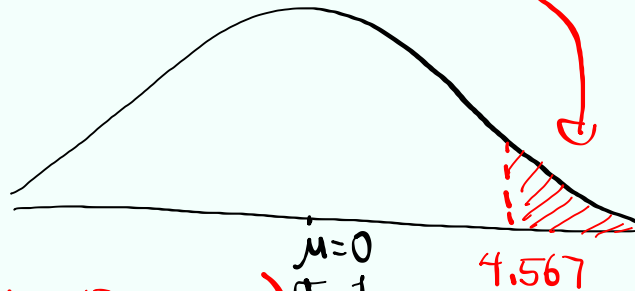
find $P(F < 1.5 \text{ OR } F > 9.5)$ with $Ndf=3$
 $Ddf=20$.



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Find twice the area to the right of

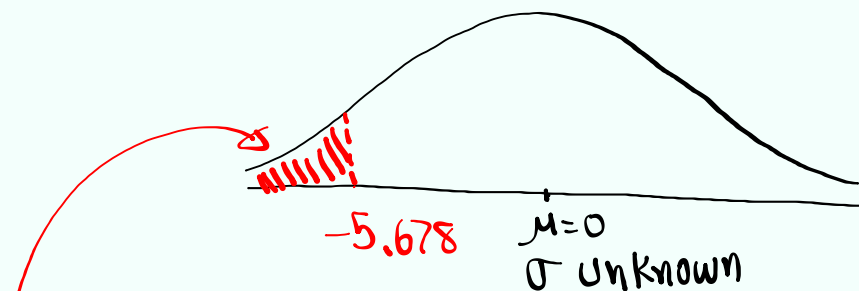
$$Z = 4.567.$$



$$2 * \text{normalcdf}(4.567, E99, 0, 1) \\ \approx 5 \times 10^{-6}$$

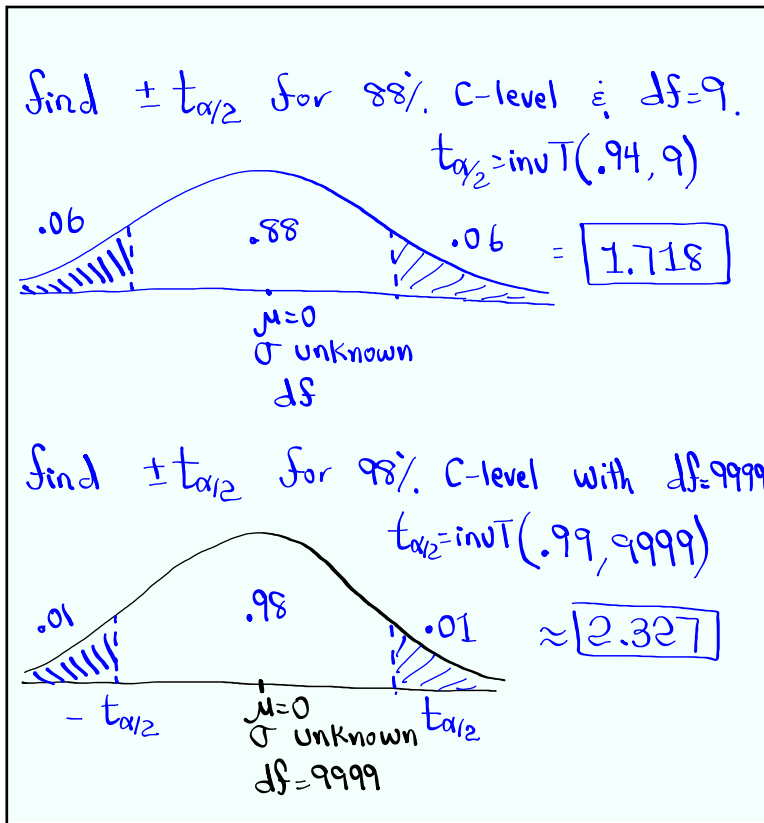
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find twice the area to the left of
 $t = -5.678$ with $df = 15$.



$$2 * \text{tcdf}(-E99, -5.678, 15) \\ \approx \boxed{4 \times 10^{-5}}$$

Nov 18-2:28 PM



Nov 18-2:02 PM